Using fractals to understand the role of entropy in complexity science: an examination of the MANA combat model

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Abstract

A new cellular automaton combat model called MANA is examined for evidence of behavior that produces fractal data. Examination of the temporal distribution of information flow (contact reports) shows clear fractal structure. This appears to result from the reaction of the model entities to the growth of disorder. The personality rules of the entities react in such a way that fractal distributions are formed, or at least, this appears to be one way in which the forces can mitigate the impact of disorder. Interestingly, while in such a formation, the entropy of the battlefield is dependent on the scale at which it is examined. We speculate that this says something fundamental about the dynamics of complex adaptive systems. It is hypothesized that such formations in a military case effectively act to isolate the highest level of command from the lowest. For the model runs examined, disorder can often grow for one force to the point where it can no longer maintain a fractal-like distribution. In this case, the distribution tends to become uniformly random, and the force appears to be finished as an effective unit.

1. Introduction

In an earlier report[1], we presented a study on the application of fractal methods to describe cellular automaton combat models. That study was in turn motivated by a paper[2] that argued that the complex nature of combat was likely to result in fractal-like distributions for many measurable aspects, for example, distribution of forces, distribution of casualties in time, distribution of radio traffic etc.

There certainly seems to be evidence, as reported by Richardson[3,4], Dockery and Woodcock[5], and Roberts and Torcotte[6], which supports the idea that combat statistics obey fractal-like power laws.

For example, the number of battles on the Western front after Normandy where casualties exceeded some level, *C*, obeyed a power-law distribution dependent on *C*, i.e.:

$$N_{\text{casualties} > C} \propto C^{-D} \tag{1}$$

where $D \approx 1$ in this case[5].

Our most recent studies were primarily intended to address some concerns with our original report. Foremost of these were the extent to which the distributions presented could be called fractals. Clearly, in some cases, the fractal "scaling range" was quite short, and it might be questioned whether this qualifies as a fractal. In particular, the use of fractals to describe the distribution of a small number of entities can be quite tenuous, simply because they do not provide much of a pattern to analyze.

We wish to make it clear that we simply used fractals as a tool to describe the distributions of the measured signals. We do not mean that the data are literally a fractal, or that some fractal "process" created the distributions. Importantly, we will see in this paper that the fractal structures hinted at in the earlier work become much more evident when applied to high-traffic-rate data, such as contact reports.

2. The MANA model

For this report we introduce a new cellular automaton combat model, called MANA (Map Aware Non-uniform Automata). MANA is different from traditional CA models, in that it allows a range of global interactions as well as local.

MANA, at least superficially, bears a strong resemblance to the widely distributed ISAAC model[7,8]. In each model, the automata's behaviors are governed by sets of parameters that determine their propensity to move towards friendly or enemy units, and towards an objective. A further set of parameters act as conditional modifiers to this process. For example, an "advance" parameter prevents automata from moving toward their objectives without a minimum number of friendly units within sensor range. A final set of parameters describes the basic capabilities of automata, such as weapons range, sensor range, movement rate, etc.

MANA differs from ISAAC in that it allows a greater range of triggers to cause a change in an entity's personality. For example, a contact with an enemy, a shot being fired, reaching a goal point, and becoming injured may all change an automaton's personality.

MANA also places greater emphasis on global interactions, by providing each side with a "memory map", which is used to mark the locations of enemies. The entities thus react to both enemies that they see, and enemies they "remember".

MANA entities use a penalty function to rank possible moves, based on their personality rules. If several moves have a similarly low penalty, a move is chosen at random from the best moves. A "movement precision" parameter sets how wide the margin should be for accepting similarly good moves. By introducing this randomness we are notionally representing small differences in the personality, or "mood", of the automata.

The impact of these features produces much more realistic looking behavior over longer periods of time than was possible with the early versions of ISAAC.

In this report, two cases will be examined, for each of which there are 40 automata on each side:

(i) All shooters are able to see and shoot at any enemy, with no maneuvering occurring (we refer to this as the "Lanchester case", due to the similarity to the stochastic version of the widely used Lanchester model of combat[9]).

(ii) Each force is attracted towards a goal behind its enemy. Initially, neither force is in contact with the other. No automaton may advance without the presence of at least four friendly units within sensor range. If in contact with the enemy, then 8 friendly units are required to be within sensor range to advance. If an automaton is unable to advance due to lack of support, it will retreat. Once out of enemy contact, it is possible for retreating automata to "regroup", since they then only require 4 nearby friends to advance again.

Unfortunately, the details of the MANA model and the parameters used are too complex to discuss in any more depth in this report. However, the authors will provide copies of the model and the data examined here on request. Additionally, the model, including the scenario sets used here, is available to be downloaded on the Website (to be determined).

These two cases provide an interesting contrast. The first case represents "robotic" fighters, who will fight to the death. The second case is designed to be vulnerable to the effects of disorder. In reality, troops do not fight to the death, but will generally fight provided there is sufficient order within their ranks. Real armies do not necessarily behave in the way we have modeled the forces here, but undoubtedly there are similar structures within armies which once disrupted cause them to fall into aimless retreat (an example is the collapse of the German armies in Western Europe during World War II, as described in US Gen Bradley's book[10]).

3. Fractal nature of MANA model data

The main purpose of this paper is to present new data that expands upon these previous findings. Rather than examine the distribution of casualties in time, the emphasis shifts to the rate at which information is being gathered about the enemy.

Define the number of pieces of information one force has about the opposing force at time step j as:

$$I_j = \sum_{i=1}^{N} n_i \tag{3}$$

where n_i is the number of enemies the *i*th automaton in a force of N automata sees. Thus if there are M enemy automata all within detection range of N friendly automata, $I = MN$ (note that this measure allows multiple counting of contacts).

Since the number of pieces of information about the enemy is much larger than the potential number of casualties, this time series is not so sparsely populated, and hence easier to test for evidence of fractal distributions.

The actual quantity we shall use for this analysis is $\left|\Delta I_j\right| = I_{j+1} - I_j$, which we will view as a sort of "informational equivalent" to attrition.

Figure 1 shows $\left|\Delta I\right|$ as a function of time for the two cases described in the previous section. In order for data to be fractal, it must exhibit temporal correlations that obey power laws. Figure 2 plots the power spectra for both cases. The spectra were obtained by taking the first 128 data points from each run, and splitting them into two 64-point data sets. Then, 16-point spectra were obtained from each set. This was done for data obtained from 20 different runs. The 40 spectra obtained were then averaged to produce a single spectrum for each case.

While the spectrum for the Lanchester case has a power-law slope of nearly 0, case ii obeys a power law with a slope of around – 2/3. Thus the latter case shows potential to be described by fractals.

To explore which type of fractal model might be used to characterize this data, the time series were analyzed for multifractal behavior. This was done by using "multiscaling" analysis. A multiscaling field has statistical moments that depend on the resolution at which the field is examined[11], i.e.:

$$\left\langle \left|\Delta I_j\right|^q \right\rangle \propto \left(\frac{t}{T}\right)^{-K(q)} \tag{4}$$

where $\left|\Delta I_j\right|$ is the attrition rate at the *j*th time step, the angled brackets represent an ensemble average, t is the temporal resolution at which the distribution is being examined, T is the "outer" scale of the scaling range, and $K(q)$ is a non-linear function of the order of the statistical moments, q.

Figure 3 shows how the second-order moment varies as a function of resolution for each case. Case ii displays a much steeper slope than case i, and makes a slightly better straight line. This suggests that case ii exhibits more convincing multiscaling properties than case i. The $K(q)$ function for case ii is shown in Figure 4. The function is non-linear in q, at least up to values for q approaching 3.

The most interesting result obtained from these data is shown in Figure 5. Case ii displays a distribution that at least partly obeys a –1 power law. This result is quite similar to the observed power law for the after-Normandy World War II data discussed in the introduction. This is particularly so when one might consider each entity to represent a battalion, say, and that $|\Delta I|$ is somewhat representative of the concurrent level of casualties, since more contacts imply more engagements. Though we are not comparing like with like, the data presented here provides tangible evidence that cellular automaton combat models can produce power laws extremely similar to those observed in real combat data.

4. Entropy on the battlefield

Case ii was designed to be sensitive to disorder. It may be useful to think about the disorder (entropy) of the distribution of the forces in terms of the change in the level of knowledge a commander has about his/her opposition as entities within the opposition force are detected.

We assume that at some starting point, each force is systematically deployed, so that knowledge of the location of one entity effectively implies knowledge of the complete force. Such a system is in a state of maximal order, since all positions of entities are known with 100% probability.

If combat occurs, the local reactions of entities will cause these formations to become distorted. The existence of a power-law spectrum for the $|\Delta I_j|$ data, which was seen in the preceding section, suggests that the shapes into which the formations evolve also have a special kind of distribution, since the power law implies that the data is temporally correlated. Specifically, detection of a given entity in a certain time step implies a greater chance that a second entity will be detected in the following time steps than would have been the case if the enemy entities were randomly distributed. Thus, knowledge of the position of one entity increases knowledge about the battlefield disproportionately.

A suitable spatial distribution to produce such behavior is a hierarchy of clusters within clusters, conveniently described by a fractal. Similar analysis to that in our earlier paper[1] could be presented here to demonstrate that the distributions of the automata in these runs possess a fractal scaling range. To save space, we will not represent such an analysis here, but leave further examination of this issue for further research. However, it should come as no surprise that automaton models produce fractal distributions, given the well-known results of Bak and Chen[12,13].

An interesting consequence of a fractal-like distribution of the forces is that the entropy of the battlefield is now dependent on the scale at which it is examined, i.e.:

$$S \propto l^{f(D)} \tag{5}$$

where S is entropy and D is the fractal dimension of the distribution of the forces.

If we associate different sized regions of radius l with regions of responsibility for officers of differing rank, then we can gain a feel for how the disorder created when the two forces clash affects the chain of command.

The highest level of command has knowledge of the battlefield that is much more certain (lower apparent entropy) but much less detailed than the lowest level of command. The idea that entropy of a complex adaptive system is dependent on the scale it is examined at does not appear to have been considered in previous investigations into the use of entropy as a battlefield measure, such as Rodrigues[14] and Barr and Sherrill[15].

It seems quite likely that this situation says something fundamental about the nature of the dynamics of complex adaptive systems. It may lead us, for example, to construct a phenomenological view of warfare:

- During an engagement, the forces involved will begin to become disordered.
- The personality rules of the automata respond to the growth in disorder. The formations distort into shapes more easily describable by fractals than by basic Euclidean shapes.
- Adapting a fractal formation increases disorder (and hence entropy), but does not maximize entropy. Furthermore, the entropy of the battlefield becomes dependent on the resolution it is examined at.

There is a final point:

- If the command hierarchy is fragile, the fractal dimension of the distribution will continue to change, until in the limiting case the distribution becomes uniformly random.

This last point is not immediately obvious from the discussion above, but can be easily illustrated using a variation of case ii. Here we allow the Blue force to advance in the presence of the enemy with only four friends in support (rather than 8). The interesting thing about this is, even though this force is now effectively "braver", it performs consistently much worse than the Red force. The reason is that the Red force drops back to find enough support (i.e. 8 friends) to make a stand. Initially, it appears Blue is driving through Red. However, when Red finds sufficient support, its forces are concentrated to the extent that it has more firepower available locally than Blue. The different degree of concentration of each force can be described in terms of each force adapting a formation with a different fractal dimension (as a result of their differing personality rules).

Figure 6 shows the evolution of this case. The formation into which Red evolves is consistently superior to that of Blue, yet has no consistent Euclidean shape. One suspects then that the Red and Blue forces evolve into fractal shapes with on-average quite different fractal dimensions. Red is then able to cause a catastrophic amount of disorder to Blue. At this point, there is no mechanism to prevent the disorder continuing to grow. The automata shown in the figure simply continue to move apart. When enough runs are examined, it becomes clear that the final Blue distribution tends to become uniformly random on average, hence tends to a fractal dimension of 0.

5. Discussion and Conclusions

The data produced by the MANA model and presented here clearly displays fractal properties. At least one of these properties is extremely similar to historical observations.

We have identified one condition that allows a combat model to be able to produce power law data. This is that the model should be sensitive to disorder. But for this MANA scenario, the creation of entropy of the distribution of the forces does not increase incrementally, unlike other schemes for describing battlefield entropy, such as Rodrigues[14]. Rather, it appears to play the role of a non-linear feedback for the model, and leads to a "fractalization" of the distribution of the forces. It is interesting to speculate whether such behavior is a necessary condition for emergent behavior. Perhaps a useful definition for emergent behavior is a system that displays such "fractalized" behavior.

Using ideas based on fractal geometry, it was possible to construct a convenient framework for understanding how a military unit copes with disorder. We hypothesize that disorder is created at the lowest levels (individual soldiers) and spreads up the chain of command. If the unit has no robust command structure, or that structure is placed under too much stress, then the unit literally flies apart (into an uniformly random distribution). If not, then the interaction between forces distorts the original configuration into patterns more appropriately described by fractals than by lines and columns. The properties of this distribution are that entropy (disorder) is neither minimal nor maximal, and depends on the resolution at which the system is examined.

This can be seen in terms of the MANA model runs described here. At the highest level is the information represented by the overall commander's "plan", i.e. the initial distribution of forces and the goal applying across the entire grid. The intermediate command level is represented by the behavior rules that require automata to support each other in advancing. These rules apply on scales of tens of cells. The lowest level of command is represented by random variations in the initial positions of the automata and the random element in the movement algorithm. These only apply to the cells immediately adjacent to the automata.

The idea that disorder starts at the bottom and works its way up to the top makes a nice analogy with turbulent cascade dynamics. For that case, phenomenologically, energy is introduced at the largest (forcing) scale, and "cascades" down to the molecular dissipation scale through a series of eddies of ever decreasing size. The military analogy is that order is created at the highest level, but destroyed at the lowest level, with a series of intermediate levels acting as barriers between the two. If so, it suggests that the lowest ranks may most heavily influence the quality of the command structure.

This leads us to construct an alternative view of warfare. The most potent force is the one with the best command structure. The objective of war is to create enough disorder to unravel the opposition's command structure. Weapons are one instrument for doing this. A laterally minded commander might well employ several other methods, such as maneuver and shock.

This offers an explanation as to why apparently very similar armies perform dramatically differently on the battlefield. Perhaps a historical example of this is the defeat of Darius III by Alexander the Great. Both sides were armed with similar weaponry, and Alexander's forces were significantly outnumbered. Yet Darius' forces fell to pieces in the heat of battle. This is strongly suggestive of a lack of cohesion in

Darius' army, so that it was vulnerable to the creation of disorder within its ranks. Although Darius' army no doubt had a hierarchy, the question is whether Darius' officers were competent enough to prevent disorder spreading up the command chain.

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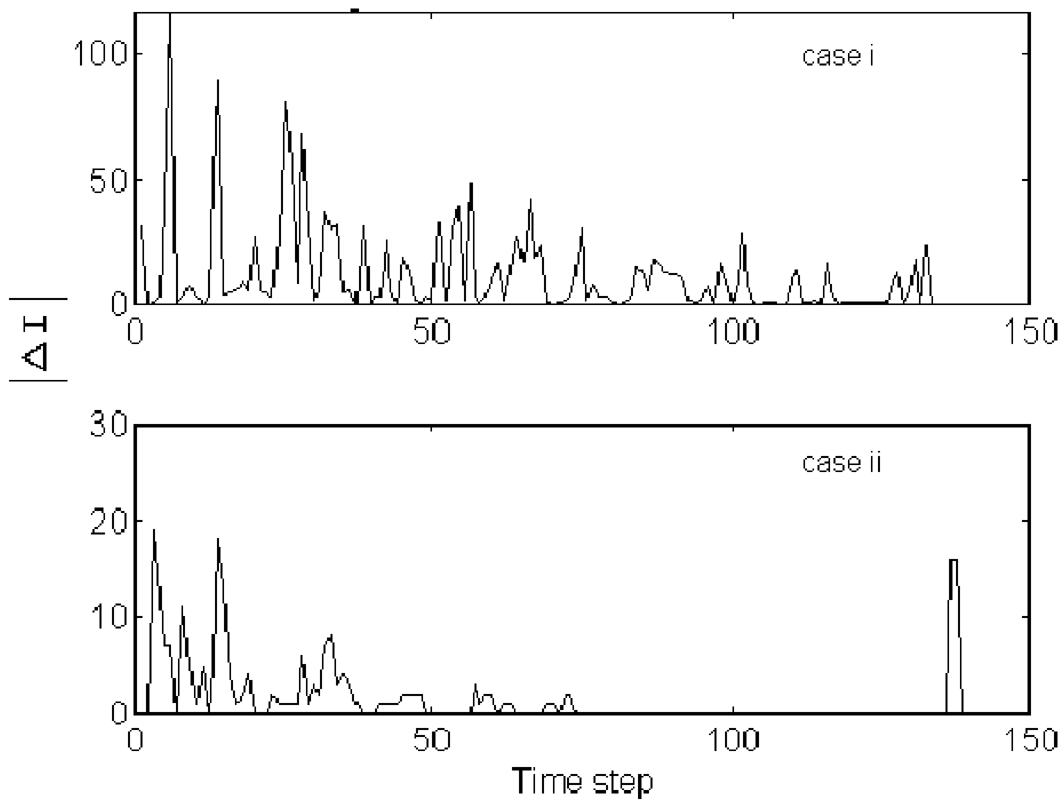

Figure 1: Variation of $|\Delta I|$ as a function of time for cases i and ii.

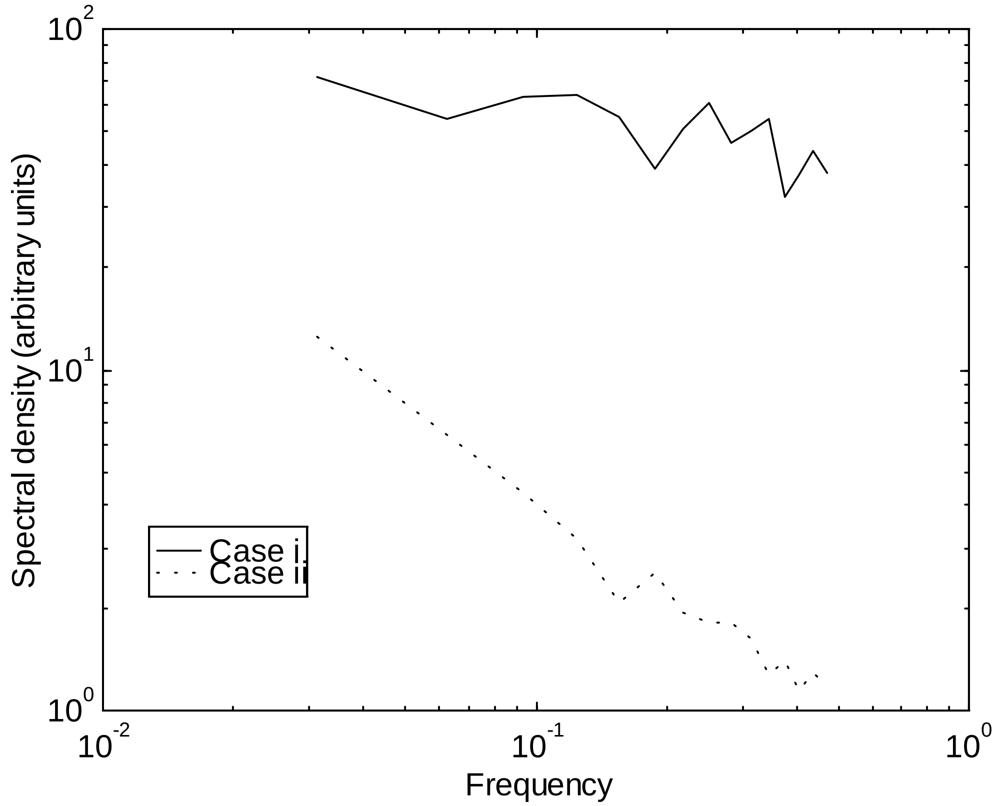

Figure 2: The power spectrum for each case.

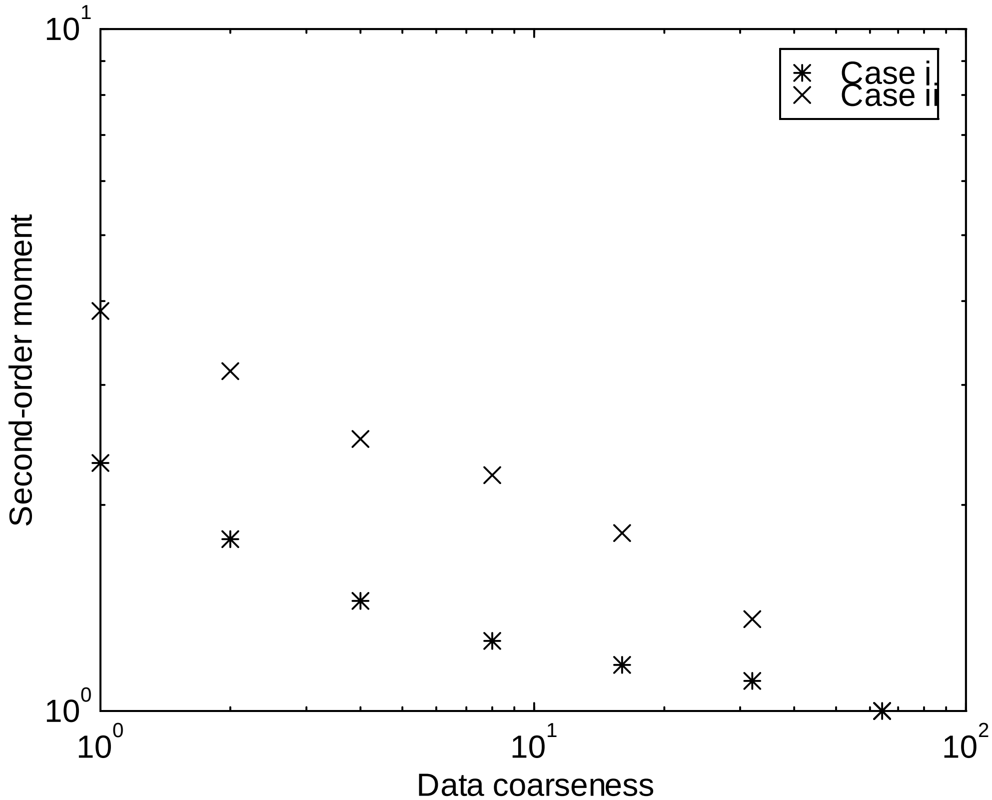

Figure 3: Value of the second-order moment as a function of the resolution of the data.

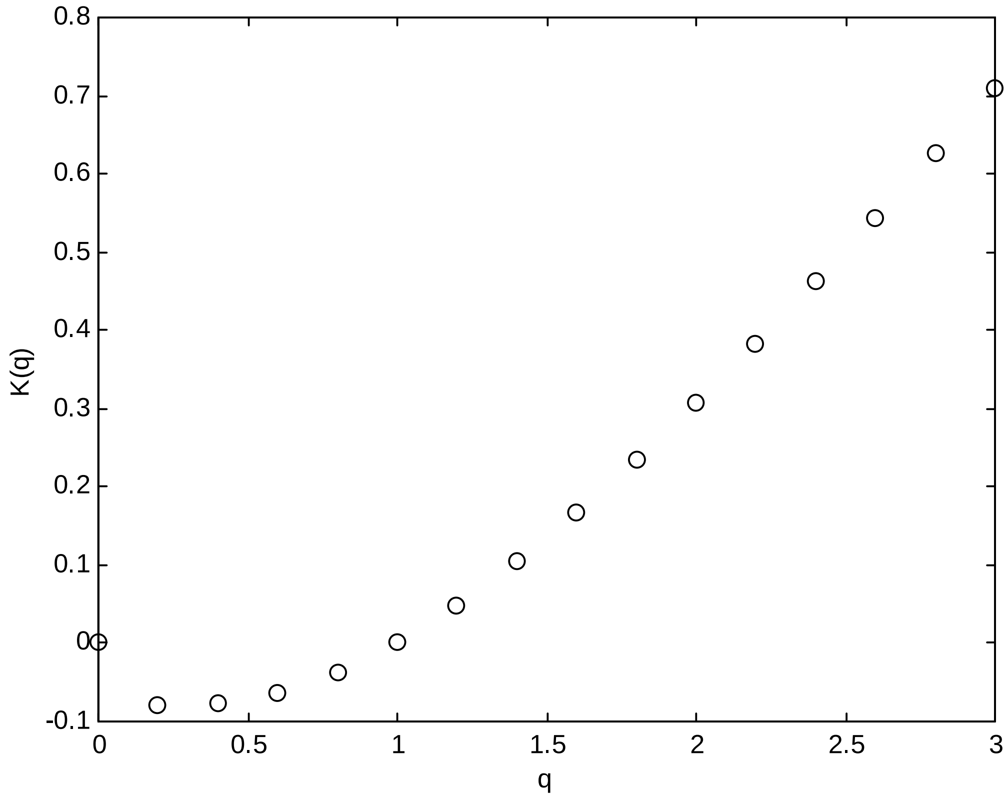

Figure 4: The function $K(q)$ for case ii.

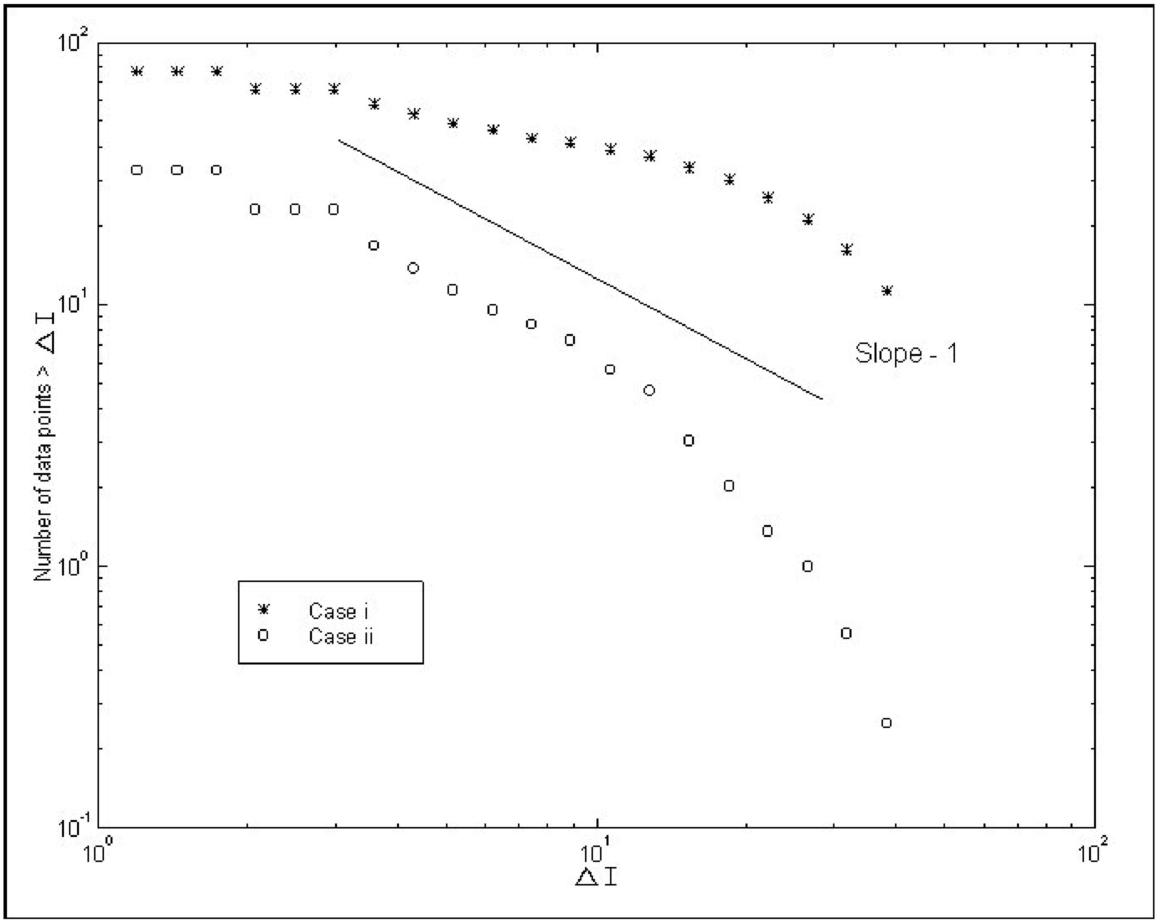

Figure 5: Case ii shows a –1 power law distribution for the number of data points exceeding some value of $|\Delta\ I|$.

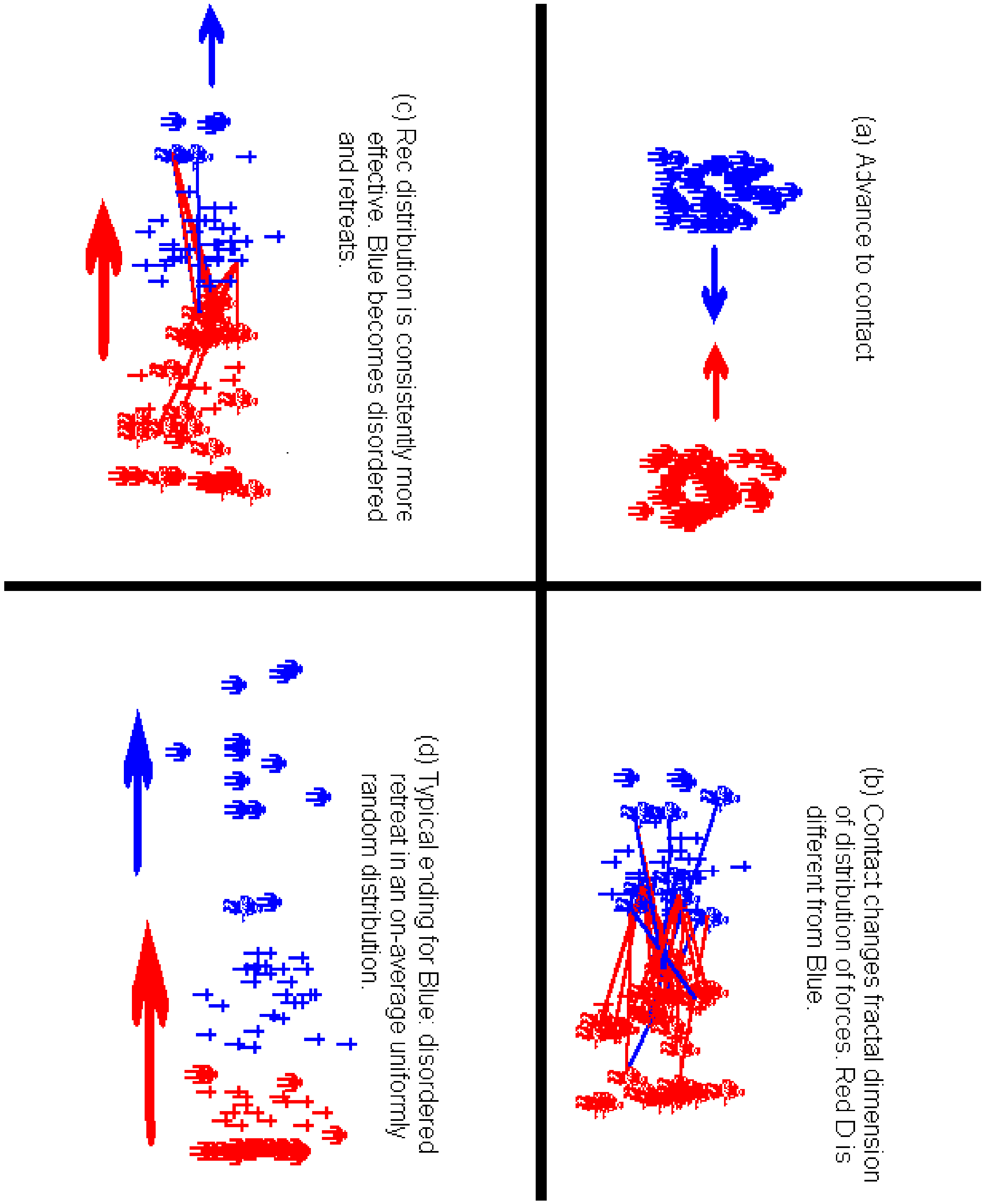

Figure 6: A MANA model run illustrating the transition from minimal to maximum entropy for the Blue force, via an intermediate “fractal” stage.